Optimal Detection of OOK Signals over Free Space Optical Channels

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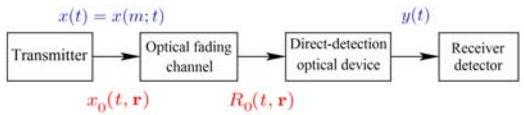
Introduction

We consider a model of a free-space optical link subject to log-normal fading due to atmospheric turbulence. We investigate a receiver structure with output modeled as a train of Poisson pulses (with rate dependent on optical input power) in Gaussian white noise. Under the assumption of on-off-keying (OOK) modulation of the optical signal at the source and knowledge of the fade at the receiver, we derive an optimal detection scheme for minimizing the probability of detection error.

Optical Fading Channel

- Atmospheric turbulence, caused by differential thermal conditions in the troposphere, results in random variations in refractive index at optical wave lengths.
- A light beam propagating through the atmosphere suffers fading with random temporal and spatial fluctuations of amplitude and phase.
- Model for an optical fading channel:



- Transmitted optical signal:

$$x_o(t,\mathbf{r}) = \sqrt{x(t)}A(\mathbf{r})\cos(2\pi f_0 t) \ ,t \geqslant 0$$

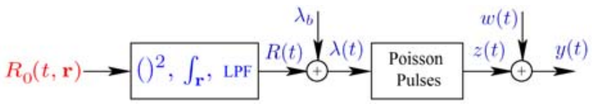
- Optical channel output:

$$R_o(t,\mathbf{r}) = \tilde{K}(t,\mathbf{r})x_o(t,\mathbf{r}) \approx \tilde{K}x_o(t,\mathbf{r}) \ ,t \geqslant 0$$

where $\tilde{K}$ is a log-normal fade.

Direct Detection Optical Device

- The output of the optical device is a train of Poisson pulses with rate dependent on short-time average power.
- The optical device includes an electronic amplifier which introduces thermal noise.
- Model for the optical device:



- $R(t) \approx Kx(t)$, where K is log-normal.
- $\lambda(t) = R(t) + \lambda_b$, where λ_b is dark current rate.
- $\{z(t) \ ,t \geq 0\}$ is process of Poisson pulses of rate $\lambda(t)$

where

$$z(t) = \alpha \sum_{k=1}^{N(T)} g(t-\tau_k)$$

with $g(\cdot)$=a unit area pulse of duration $\varepsilon << T$.

- $\{w(t) \ ,t \geq 0\}$ is white Gaussian noise with $psd = N_0/2$.

OOK Optimal Detection: Problem Setting

- Assume the fade value $K = k$ is known at the receiver.
- The optical signal is on-off keying modulated i.e. $x(t) = 0$ or $x_0 > 0$ during $[0,T]$. Thus, the rate is $\lambda(t) \equiv \lambda_0 = \lambda_b$ or $\lambda(t) \equiv \lambda_1 = kx_0 + \lambda_b$.
- Based on the observation of $\{y(t), 0 \leq t \leq T\}$ find a decision rule for detecting the OOK signal (i.e. 0 or x_0), which minimizes the probability of detection error.

Optimal Detection Rule

- The optimal detection rule is a “threshold test” i.e.

$$\hat{x} = \begin{cases} x_0 & ,L \geq \eta \\ 0 & ,L < \eta \end{cases}$$

where L is the likelihood function and η is a known threshold.

- The likelihood function is obtained as

$$L = L(\{y(t) \ ,0 \leqslant t \leqslant T\}) = e^{-(\lambda_1-\lambda_0)}\frac{\phi_1(T)}{\phi_0(T)}$$

where $\phi_i(T), i=0,1$ can be approximated through the following integral equations:

$$\psi_i(t) = 1 + \lambda_i \int_0^t \gamma(t,s)\theta(s)\psi_i(s)ds$$

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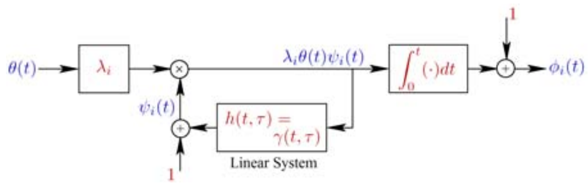
and

$$\theta(t) \triangleq \exp\{\frac{2\alpha}{N_0}\int_0^T [x(s) - \frac{\alpha}{2}g(s-t)]g(s-t)ds\}$$

$$\gamma(t_1,t_2) \triangleq \exp\{-\frac{2\alpha^2}{N_0}\int_0^T g(s-t_1)g(s-t_2)ds\}$$

Scheme for real-time computation of $\phi_i(T), i=0,1$:



Planned Work

- Performance evaluation under various assumptions on the fade.
- Joint detection and control based on optical signals.